Going the distance: a lesson study on deriving the distance formula

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Abstract

Purpose – The purpose of this paper is to determine the challenges that Filipino Mathematics teachers face while developing students' ability to derive the distance formula; allow teachers to collaboratively formulate a lesson designed to address the challenges they have identified from their own practice; and determine how successful the lesson was and how it can be improved so that other teachers dealing with similar difficulties may be able to implement it. The teacher-participants employed Lesson Study (LS) as an approach to improving pedagogical practice. Data were based on the pre- and post-lesson discussions and individual reflection papers of the teacher participants.

Design/methodology/approach – An action research methodology through LS approach was employed by the teacher participants. Data were based on the pre- and post-lesson discussions and individual reflection papers of the teacher participants.

Findings – Based on the post-lesson discussion, the teachers agreed that the process of creating a lesson that seeks to develop the students' ability to derive formulas are crucial to building understanding of the underlying mathematical concept. Also, teachers' participation in LS was found to have been insightful as it developed in them a greater appreciation towards establishing a professional learning community that is directed towards examining problems that concerns majority of the teachers involved.

Originality/value – Research in Philippine education has recently seen the increasing interest in LS as a potent pedagogical practice. Nonetheless, papers that report on LS practice in the local context remains to be lacking. This study contributes to the development of this research area and raises the need for Filipino Mathematics teachers to engage in both LS and action research to generate knowledge from their experiences.

Keywords Lesson study, Action research, Constructivism, Mathematics education

Paper type Research paper

Introduction

Critical thinking and problem solving are at the core of the mathematics education program in the Philippine K to 12 Curriculum – a curriculum that has restructured the learning system and standards for the Filipino learner by fully embracing a constructivist perspective to education (Department of Education, 2013). The greater emphasis on developing these abilities among learners is apparent in the type of learning competencies and standards outlined in the curriculum guide for mathematics. In comparison to the curriculum it replaced in 2012, the current curriculum expects students to develop skills by constructing and refining their own understanding of mathematical concepts with the guide of the teacher. As such, while students whose learning experiences were designed to follow the previous curriculum were exposed to applying mathematical formulas without being expected to derive these themselves, those under the current curriculum are challenged to use and hone their critical thinking and problem-solving skills to walk the same path as the mathematicians who proposed the formulas with the guidance of their teacher.

In fact, the Filipino learner is expected to derive the formulas for the area of a rectangle and a square in the third grade. This is followed by 17 other learning competencies that aim at either deriving a mathematical formula, the laws of exponents or relationships in the

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parts of geometric figures. These competencies are distributed across the other grade levels until the end of the tenth grade.

Emphasis on training students' ability to derive the actual formulas used in math has become a concern for teachers in Dolores National High School for two reasons which were raised in one of the roundtable discussions regularly conducted by the school's mathematics department. First, despite our continuous efforts to design lessons that reflect key principles of social constructivism, many of our learners struggle with taking a more active role in their learning. These are learners whose formative years with mathematics were marked by a more behaviorist pedagogy that embraced traditional learning – a classroom environment marked by exposition and greater attention to procedural fluency at the expense of other key strands of mathematical proficiency. Second, some teachers find lessons on formula derivation to be a source of frustration because it demands more effort to design plans that allow for discovery instead of deduction and more time for students to generate workable results. The time spent on allowing students to construct their own understanding is believed to take away time that can be spent on developing their abilities to use these formulas. However, student performance on tests and accounts of teachers have led us to believe that taking away the opportunity for the learners to discover patterns leading to important formulas prior to application may be the cause of weak conceptual understanding. We are inclined to believe that this challenge in instruction is not uncommon and may as well be a concern for other math teachers in other schools in the Philippines.

As part of the research, teachers collectively examined their own practice in dealing with competencies that center on formula derivation, that is, how this type of learning competencies is carried out in the classroom and what common difficulties teachers and learners have. Two dominant practices have emerged. Some teachers expressed their belief that the derivation of formulas often contribute to their students' math anxiety and undertaking such task before application of the formula often contributes to several other problems that are linked to their productive disposition. Their students' continued struggle with this process that made these teachers decide to proceed directly to the application of formulas and skip the whole derivation. The other fractions of teachers who discuss derivation in their classes do so through lectures and without expecting participation from students. Moreover, they inform students that "the focus of the topic is not necessarily the derivation but how the formula is applied." None of the teachers, prior to the conduct of this research, has used any exploratory approach in deriving formulas.

Due to these practices, learning competencies that aim at students' ability to derive formulas are not necessarily met by students and therefore do not adhere with the intended competencies set forth by the Department of Education.

This paper discusses the conduct of a series of activities that tested out the feasibility of a lesson targeting the derivation of the distance formula which uses guided discovery approach.

More specifically, the research activity described in this paper was done with the following objectives: assess the effectiveness of a lesson on the derivation of the distance formula that employed a guided discovery approach to teaching; develop and evaluate innovative instructional materials on the lesson study (LS) topic; and expose the math teachers of Dolores National High School to LS.

The study was conducted by 11 secondary school math teachers of Dolores National High School in Eastern Samar, Philippines. The LS group also included the school principal, head teachers of various subject departments and two other school principals who previously taught mathematics. All three principals took the role of knowledgeable others.

Theoretical background

According to Mumford (2014), an equation can be thought of as a quantitative metaphor, one which if not internalized, will condemn a person to dredge up isolated rules every time similar situations prompt the need for it. Formulas exhibit relationships that exist among variables – a relationship that states in mathematical form how one variable is affected by one or more variables. Equipped with a formula, a student has the ability to solve problems in relation to context for which the formula works without going through the tedious process of deriving the formula repetitively based on the problem.

However, relaying the formula to the students may be considered insufficient as it does not provide the opportunity for the class to understand how the formula came into being and in what way it makes sense as a means to solving problems of a specific type.

Ostler (2011) explains that the focus on using formulas has become so common in secondary mathematics instruction that we have underestimated the value of contextualizing the processes that allows them to exist unless teachers truly consider the innate limitations of the way we teach procedural fluency, we will perpetuate the belief that procedures alone represent what constitutes good mathematical learning and what makes good mathematicians.

Students continue to struggle with adapting their learning beyond the rudiments of their textbooks because the most common instructional method used in secondary mathematics classrooms is highly structured and is based primarily on the semiotic processes so commonly illustrated in mathematics textbooks (Watanabe, 2007).

Formula derivations do not typically emphasize the development of procedural fluency. For this reason, they are often viewed as inefficient or even superfluous exercises in the classroom. The counterpoint, however, suggests that vaguely defined tasks are exactly what allow strategic and adaptive reasoning to occur in a mathematics lesson (Ostler, 2011).

When teachers scaffold students to work in collaborative group situations their opportunities for interaction and engagement in a range of key mathematical practices increases (Gervasoni *et al.*, 2012). When problem-solving groups are employed by the teachers and where students of varying mathematical expertise are grouped together to engage in opportunities to "talk and do" mathematics (Hunter, 2010). In fact, aside from Hunter's, another study by Askew (2011) provides evidence that grouping resulted in students achieving more than they would independently.

With appropriate guidance from teachers, students can build a coherent understanding of mathematics, and that their understanding about how the symbolic processes of mathematics can evolve into increasingly abstract and scientific reasoning (Romberg, 2000). The type of questions teachers asked influenced the nature of the students' responses; however, developing appropriate questioning skills has its challenges and teachers' questions are often limited to recall or seeking clarification (Muir, 2009).

Based on ideas of Vygotsky's theory of constructivism, the teachers planned a problem-based lesson on the topic taking into consideration students' previous knowledge of right triangles and the Pythagorean Theorem. In constructivism, learning is contextual. Second, knowledge is needed to be learned (Hein, 1996). These principles point out the idea that the human being learns through associating the idea presented to him to his prior knowledge. Any effort to teach must direct the learner into the subject with a basis on that learner's previous understanding.

Lesson study as method

In order to develop and assess a lesson plan that targets the students' ability to derive the distance formula, the researchers employed a LS approach.

LS is a teacher professional development approach. It is a cycle composed of planning, doing and seeing (Fernandez and Yoshida, 2004). In every LS cycle, the result of the discussion from the previous cycle is used to improve the lesson so that every cycle leads to a better lesson. This paper reports the initial cycle of the LS conducted by the group.

The paradigm (Figure 1) that follows is adopted from a study by Ebaeguin and Stephens (2014) which shows the three component phases of the LS cycle and also provides description of the activities that we did for each phase.

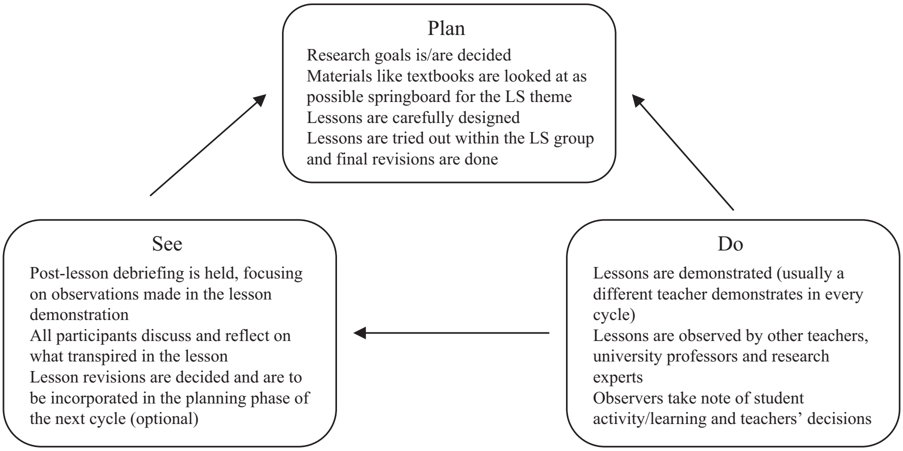

Figure 1.
The plan-do-see model of lesson study

Planning stage

Teachers who were part of the LS group collaboratively planned the entire process. This planning stage involved a series of brainstorming activities that determined what specific problems are most prevalent and needed priority, then sorting out which of these problems would be most convenient in that its suggested solutions that maximizes resources and the time available.

Originally, the most important criteria decided was that the topic must be in the tenth grade and is one which students in the past have found to be difficult. A review of the curriculum guide directed the group's attention toward the need for the learners to derive the distance formula. Applying the formula was never seen as a problem for most students but asking students to explain how the formula is derived is one that became a concern.

Another criteria which the LS group looked into was the significance the results of the study would have. The group agreed that developing a lesson centered on deriving the distance formula encompasses the topic itself as it would also provide for a more general knowledge on how teachers can target lessons that involve derivation of mathematical formulas. By focusing the LS activity on this specific topic meant that teachers would be able to test the success of employing discovery into a topic that asks students to use induction.

The planning stage also involved a training of all teachers involved about the LS cycle. Fortunately, three of the teachers had prior experience in the process gained through seminars, trainings and actual LS undertaking.

Identifying the specific roles the teachers played was also an important aspect of this stage. This included assigning the demonstrating teacher, those who were in charge of selecting and coordinating with prospective knowledgeable others, and those who did equally important tasks such as helping in the preparation of the intended instructional materials and documenting the activities.

A study of the available related literature, as previously mentioned, supported the importance of lessons allowing students to derive mathematical formulas through guided discovery. Teachers who handled the tenth grade also convened and discussed the following: what was the usual practice in the discussion of the distance formula? And what difficulties did the students have in dealing with the formula or deriving it, if it was a part of the class discourse? The outcome determined how the lesson was to be planned.

Principles of constructivism supported the decision of utilizing a problem-based lesson aiding the development students' knowledge through inquiry.

The lesson underwent two major revisions. The initial draft was presented to the whole group by the assigned demonstrating teacher. Comments and suggestions were taken into account for the first revision. The revised lesson was presented to the group for the second time and was again revised for the final draft.

In terms of students, the LS group sought the participation of a class of 40 students. Majority of the students were identified to have average mathematical proficiency levels and some were performing poorly in the subject.

Doing stage

The second stage was the actual demonstration teaching. Three knowledgeable others have been invited to participate including the school principal and two other principals who previously taught mathematics. Prior to the demonstration, these knowledgeable others and several other auditors have been briefed regarding their role in the activity and the epistemological orientations which include the review of literature, the previous practices and the objectives of the LS.

Three teachers have been asked to document the activity during the demonstration. One of these took pictures, another was in charge of taking notes and the third facilitated the video recording of the lesson implementation.

Seeing stage

A post-lesson discussion immediately followed the demonstration. The LS group reconvened to assess the implementation of the lesson. The demonstrating teacher was first given the opportunity to reflect on his experience in delivering the lesson. His time centered on the difficulties he encountered in the implementation of the plan, the problems his students had including those that he observed while interacting with the students, and his general thought on the success of the lesson based on the objectives of the LS.

Thereafter, the invited knowledgeable others were given the chance to evaluate the lesson and provide suggestions to better improve the topic. The knowledgeable others and the other observers present were given observation protocols containing guide questions that they may focus on during the activity. The purpose of the protocol was to maintain focus on the effectiveness of the lesson by emphasizing the concerns the teachers had.

The same teachers who were assigned to document the demonstration also facilitated in putting into records the salient points in the discussion while a moderator made sure that the discussion was focused and that it followed a predetermined sequence.

Results and discussion

For this unit of the paper, the results of the research practice are discussed through three parts. The first gives a comprehensive narrative of what happened during demonstration teaching and the responses of students to the questions raised. Under the second part, salient points in the post-lesson discussion are used to explain what occurred in the lesson properly with an emphasis on the areas needing improvement. Finally, recommendations are raised based on the outcome of the whole set of activities.

The research lesson

Two activities have been developed by the LS group with each serving a different purpose. The first was a preliminary activity called "Guess whose number" in which students formed a human coordinate system and each member of the class represented a specific point that corresponds with an ordered pair. The activity was developed not only to prepare students for the more rigorous task that followed but also to activate their prior knowledge on identifying the location of points in the Cartesian coordinate plane. The decision to tap on students' competency with the coordinate system was deemed necessary since teachers had identified cases of alternate conceptions of learners relative to this. On the other hand, the second activity prompted students to work in groups of three in solving a problem described through a poem, "Do as the pointees do." In planning the lesson, the LS group wanted to try something different in the manner problems are presented to the students. Presenting the problem situation though rhymed story was an attempt at providing the class with an alternative form of instruction that they may have not encountered in their previous math classes.

In the conduct of the initial activity, students' seating arrangement was designed so that it represents the Cartesian plane with every student designated to a specific point. There were eight rows and seven columns with the same spaces in between seats. The middlemost seat was left empty and was designated as the point of origin. At the start of the activity, students in the middle column and the middle row, containing the empty seat, were asked to stand up so that the class can better understand how the human Cartesian plane looks like. These row and column were designated as the axes of the coordinate system. The aim of the game was for students in to locate as many correct points as possible by picking on a paper that contained the name of a classmate and then stating the coordinates that classmate represented or was located at.

Each of the two groups were given a couple of minutes to do the task with the students at the front taking chances one at a time. With the limited time given to them and the class needing time to adjust to how the model of the Cartesian plane works, some students were noted to experience difficulties due to confusion with how the coordinate plane's orientation. There was a clear struggle with those who took their turn first as they were the ones who needed to understand the rules first. Those who came after, struggled lesser as they began to realize the mechanics and were able to formulate a strategy based on how those that preceded them did.

In the end, seven students from the first group and five from the second got the correct answers in the task. All students were able to participate. The common error committed was interchanging the x- and y-coordinates and confusion with the signs.

A poem composed by the demonstrating teacher was used in the second activity. The poem provides a story and also instructs the students who were then grouped in threes to work with problems related to distance between two points. In the story, they had to locate three points whose coordinates were given and find how far one point was from the other. These three points, when connected, formed a right triangle. It was designed in this manner so that students may be able to suggest and use the Pythagorean Theorem to find the distance between the two points that formed the hypotenuse.

There were three problems they had to solve. First, they were asked to find the distance between two points that are aligned horizontally, thus having the same y-coordinates. Second, they had to determine how far two vertically aligned points are from each other, those who share the same x-coordinates. Finally they had to find a means to solve the distance between the first point and the last point whose coordinates both differ. When connected, the three points formed a right triangle – something that the student had to infer based from the points they plotted.

During the activity, the teacher examined the work of all groups and encouraged them to raise their hand if they needed any help working with the activity. This was how additional instruction was carried out. If the group felt that they were not having any problem at all, the teacher made sure not to interfere and allowed them to work with the rest of those in the same group. Several students exhibited the need to be given assurance so they continued to ask the teacher if what they were doing was right. The teacher responded with phrases that indicated encouragement and allowed them to examine their own conjectures. In the planning of the lesson, the LS group agreed that in the instance of students' raising questions about their own understanding, the teacher must ask questions that will help the student to appraise their thoughts and lead them into realizing the validity of their ideas. This practice is a potent exercise at developing critical thinking because it allows the student to be more responsible and independent in constructing understanding.

Whenever a group did not know how to proceed with the task, the teacher asked guiding questions as a means to scaffold students. As an example, the dialogue below was taken in verbatim:

Student 1: (*shows the Cartesian plane with the three points plotted to the teacher*) Sir, we don't understand (the question).

Teacher: (*names one student*) read this (*points out the stanza containing the question*) so that we can understand.

Student 2: (*reads the lines*)

Teacher: So, what are we asked to find?

Student 1: How far the cake shop is from the huge huuuge map?

Teacher: Correct. So how do we do that? (*No response from the group*)

Teacher: (*names Student 3*) if I ask you how far away are you from (*names another student*) how would you answer me?

Student 1: We measure.

Teacher: But how? We don't have a meter stick. Can we use another way? How about the number of students?

Student 3: Yes! (I am) three students (away) from (*name of the other student*)?

Teacher: Okay. Do the same for the points you have with you.

Notice that the teacher's responses were questions instead of direct answers.

What follows is another portion of the dialogue that occurred between another group and the teacher when the group asked for directions on how to find the distance of the third side:

Teacher: What kind of figure were you able to form (from the points)?

Student: A triangle, Sir.

Teacher: Nice. Let's be more specific, okay? What kind of triangle?

Student: Ah, sir, right […] right triangle

Teacher: Good. So you were looking for the distance between which points again (*motions them to identify the points in the plane*)?

Student: Sir, this one (*points to the location*), the map, and this (*points to another location*), Maria's house.

Teacher: So this one (*traces the segment joining the two points*), what is this called, if the whole of this is, like you said, a right triangle?

Student: Hypotenuse? Ah, okay sir, gets *ko na po* (I got it).

The dialogue above supported the idea that learners have the capacity to generate essential ideas if the teacher is able to ask the right questions – questions that provide hints at an idea but do not reveal what the teacher ultimately expects the learner to respond with.

After the allotted 25 min, the students were told to submit their outputs and prepare to discuss their answers to the rest of the class. The teacher started the discussion with an interactive reading of the poem. The teacher read the poem and asked to students to participate by saying words like "Wow!" and some other words whenever a certain keyword or phrase was read. They were also asked to complete the poem by saying aloud their answers to the questions whenever these questions were mentioned in the poem-reading. The purpose of this was to immerse the students in the whole task again and assess how they did in comparison to their peers. It was an activity that conditioned the class that, at that point, they were no longer separated in small groups, but were expected to work as one towards a generalization.

In this process, the teacher's ability to scaffold students' ideas was essential to the success of the discussion. The LS group planned the lesson in such a way that the teacher gave just the right amount of hints to direct students toward the objective. In this sense, questions that were asked needed to be vague enough so as not to give away the needed answers or make the hints too explicit, but specific enough that the students would realize how to connect the ideas they have already established.

First, some students were asked to plot the points they identified then connect them with line segments on the Cartesian plane drawn on the board. The length of the segments represented the distance between the two points. The class was then asked to explain how they went through each of the first two questions. As expected, counting the number of units one point is away from another was the popular answer. The teacher, with the knowledge that one group did a different solution by subtracting coordinates instead, asked one member of that group to show their solution on the board. The student was then asked to explain how they came up with the solution. The teacher also prompted the rest of the class to share what they thought of this solution to which everyone seemed to be satisfied with the process.

The teacher, however, saw that it was necessary to point out why, in the case of the horizontally aligned points, only the x-coordinates were used to find the distance, to which one student not belonging to the group who presented the answer explained that even if they did, the y-coordinates are the same and it would still have been zero and would not have made any difference in the answer. Since this process was not the popular one, the teacher urged the others to confirm their answers to the second question.

The most critical part of the discussion was finding the distance between the first of the three points and the last one since these were neither vertically aligned nor horizontally aligned. The teacher however counted on the fact that the students already gained knowledge about right triangles and the Pythagorean theorem which was the easiest method in getting the answer. One student coming from a group which got the correct answer was asked to explain the process their group did.

At that point, the teacher guided the class into discovering the distance formula. Using the previously established idea of subtracting coordinates, the teacher first prompted the students if they could incorporate the mathematical statements into the formula provided by the Pythagorean Theorem. A student, suggested substituting a and b in the original formula with $x_2 - x_1$ and $y_2 - y_1$, respectively, thus, obtaining a formula that imitates the distance formula. While explaining that D would be a more appropriate variable instead of c in the formula, and extracting the roots for both sides of the equation, the teacher finally rewrites the formula for distance on the board.

As a means of verification, the teacher then urges the students to verify their original answers obtained through the Pythagorean theorem by working with the derived formula. An assignment was given to the class to act as exercise in the use of the formula.

Post-lesson discussion

As how it is in LS, analysis of the lesson started with the reflections of the teacher who carried out the lesson. The demonstrating teacher took the time to identify the difficulties the students encountered and what he learned from implementing the lesson.

One of the initial difficulties the teacher had to deal with was relative to comprehension. With the poem used as part of the milieu being in English, some students who had difficulties in the language expressed their difficulty in understanding some portions of it or what it was asking them to do. In relation to this, Gervasoni *et al.* (2012) elucidate that culturally responsive pedagogy requires that teachers are aware of the literacy demands of tasks. Several studies (e.g. Bautista, Mitchelmore and Mulligan, 2009; Bautista and Mulligan, 2010; Bautista, Mulligan and Mitchelmore, 2009, as cited by Gervasoni *et al.*, 2012) have previously examined how disadvantaged Filipino students with limited English language, engage with English word problems. The studies suggest that teachers provide appropriate time for students to understand the problem situation, narrate the problems, support the narration with concrete tasks, support the understanding of structures that underpin the number operations, and encourage the use of representations that are meaningful to the students.

Also, There was a minority of groups who showed diffulties working with plotting points in the Cartesian plane. This led the LS group to believe that the motivational activity was not entirely successful in preparing some students for the task.

Of the 16 groups, 5 initially did not seem to realize any strategy in finding the distance. Of the other 11, 2 groups resorted to using a ruler and actually measuring the length of the segments joining the three points. Eight of the remaining nine groups used counting of units but three of these included the initial position in the counting. There was only one group which actually used the difference in the coordinates as a means to find the distance. In end, all groups, some with the guidance of the teacher, found the correct answers.

Only six groups were able to find the correct distance between the points which were neither aligned horizontally nor vertically. All six used the Pythagorean Theorem as a method but half of these groups needed the help of the teacher to realize this.

Finally, the teacher expressed that framing the most appropriate and effective questions for scaffolding impose a lot of challenges to the implementing teacher and should be carefully planned. All possible problems should be examined beforehand to prepare the teacher for the difficulties both the class and s/he may encounter.

On the other hand, the following suggestions and recommendations have been raised by the knowledgeable others and the other observing teachers:

(1) Incorporating poetry into the lesson was creative because it gave students more opportunity to learn the topic by presenting it through a different medium. Not only was the poem written in a light mood but it expressed a situation that was relatable thus helping in decreasing anxiety experienced by the class. This was apparent in how students freely interacted with others in the same group and how they were able to express their confusion and ideas to the teacher.

(2) Time and effort on the part of the teacher was a major concern as it was evident that arranging the seats, assigning groups, and preparing students for the activities took time. In the context of the school, it was teachers who visited the classrooms of the students. This means that the teacher does not have control over how the classroom will be arranged before the class started. There was a consensus that implementing this lesson requires the teacher to make necessary arrangements prior to the class. Unlocking of difficulties including words used in the poem that students are unfamiliar with may also be given as an assignment before the class.

(3) Activities were considered well-planned. The teacher, however, should rethink the review activity on the human coordinate plane. Since it was a game of time and students still needed to acclimate themselves to the instructions given to them, there was a fear that students would not have enough opportunity to respond because before they even get the chance to learn what is expected of them, time was up. It was agreed that lengthening the time limit would address this problem.

(4) The main activity was appropriate and effective in helping students establish concepts leading to the derivation of the distance formula. The teacher should be cautious in guiding students into discovering the distance formula as there is a thin line between asking the right questions and giving away the answers. The questions that should be raised during the discussion part after the activity should therefore be reviewed and improved.

Final remarks

Lessons that target the derivation of formulas can be complicated, especially if the tasks impose challenges to both the students and the teacher. Based from the LS conducted and the multiple discussions carried out across the process, the participating teachers believe that the study of the lesson has brought about two important ideas that mathematics teachers and researchers may look into and may conduct future investigations on. These conjectures are based primarily on the collective experiences of the LS participants reaped from qualitative data. As such, conclusions are non-conclusive but offer perspective into instructional efforts similar to that described in this paper.

First, to the members of the LS group, the active engagement of the learners who participated in the research lesson was apparent and was an improvement in contrast to the prior experiences of the teachers in similar lessons. The positive appraisal of the lesson has led the group to agree that in developing conceptual understanding of mathematical formulas, a guided discovery approach presents the opportunity for a more holistic development of mathematical proficiency that addresses not only procedural fluency but also conceptual understanding and productive disposition. Furthermore, when lessons are able to foster in students a strong foundation of a concept, teachers will be able to provide subsequent tasks that can help student progress in terms of adaptive reasoning and strategic competence. The attempt at employing such an approach was marked by challenges such as students' difficulty with expressing their thoughts and working with others to make generalizations from their individual understandings. Hence, it is necessary that the teacher is able to plan carefully the activities to include foreseeing possible conceptions and alternate conceptions students would create from dealing with tasks. Scaffolding is an essential skill that the teacher must develop for the lesson to be effective. Moreover, the constant exposure of students to activities that challenge them to think, construct ideas and refine their understanding is recommended to help them develop a culture of independence, action, and responsibility for their own learning.

Second, in thinking about their experience in doing LS, all those involved expressed their belief that LS is a good avenue for a small professional learning community to build lessons based on practice and not on general theories. Teachers belonging to the same school environment, working with learners having the same socio-cultural background most likely deal with similar pedagogical challenges. Activities that foster collaborative and active professional development exercise can help direct efforts into the most pressing problems and institute a support system among teachers. Furthermore, participating in LS potentially gives the teacher an idea of what works and what doesn't given a specific kind of learner or class. It also promotes a healthy and productive relationship amongst teachers in the school community by fostering constructive criticism and the need for professional development.

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